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THE MAXIMUM ECHO AREA OF IMPERFECTLY  
CONDUCTING DIPOLES

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## ABSTRACT

Recently, the question has often been raised as to when the effects of imperfect conductivity must be taken into account when studying the echo area of a wire scatterer such as the dipole. This paper clearly shows the conditions under which the echo area is sensitive to the conductivity.

Curves for imperfectly conducting resonant dipoles are presented that show the dependence of the maximum echo area upon conductivity, frequency and dipole diameter. The maximum echo area is defined to be the peak echo area at first resonance.

The maximum echo area is related to the penetration of the electric field into the dipole. This makes it possible to obtain a universal curve that may be used to determine the maximum echo area of a specified dipole without actual computation of its resonance curve.

**Accepted for the Air Force**  
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## THE MAXIMUM ECHO AREA OF IMPERFECTLY CONDUCTING DIPOLES

### I. INTRODUCTION

It is known that the maximum or peak echo area of an imperfectly conducting dipole is dependent on the conductivity,  $\sigma$ , frequency,  $\omega$ , and the dipole radius,  $a$ , whereas the maximum echo area of a perfectly conducting dipole is not. Hence, it would be useful to relate, in some fashion, the maximum echo area to these parameters. It is the primary purpose of this paper to obtain such a relationship.

For purposes of definition, we will take the maximum echo area to be the peak echo area of resonant short-circuited dipoles, where the dipole length is adjusted (near  $\lambda/2$ ) so as to achieve the resonant peak.

It is also worth mentioning that the echo area curve of a perfectly conducting dipole can be completely specified by specifying in turn the dimensionless parameters,  $L/\lambda$  and  $a/\lambda$  shown in Fig. 1. However, if the dipole has imperfect conductivity, a third dimensionless parameter is necessary in order to completely specify the resonance curve. We will show later that this third dimensionless parameter is a product of the conductivity, frequency and dipole radius.

Although the maximum echo area does vary with the conductivity, frequency and dipole radius, the relationship between these parameters and the maximum echo area is apparently tied to the penetration of the electric field into the dipole. For example, if the conductivity of the dipole material and/or the dipole radius is such that the electric field does not penetrate appreciably to the dipole axis (ie., the  $z$ -axis in Fig. 1), then the echo area will be nearly that of a perfect conductor as shown in Fig. 2 for the two 80 mil wires. However, if the electric field does reach the dipole axis without large attenuation, then the echo area can be expected to fall significantly below that of the corresponding perfect conductor. We will show that there is a relationship between the maximum echo area of imperfectly conducting dipoles and the amount of penetration by the electric field to the dipole axis. This is done in some detail at frequencies of 300 MHz and 3000 MHz for both bismuth and platinum dipoles. Representative conductivities of these and other metals are given in Table I.

Then, from the data for bismuth and platinum dipoles, a curve is deduced from which the maximum echo area may be very closely estimated for dipoles of other materials operating at various frequencies without recourse to the more involved process of actually computing resonance curves.

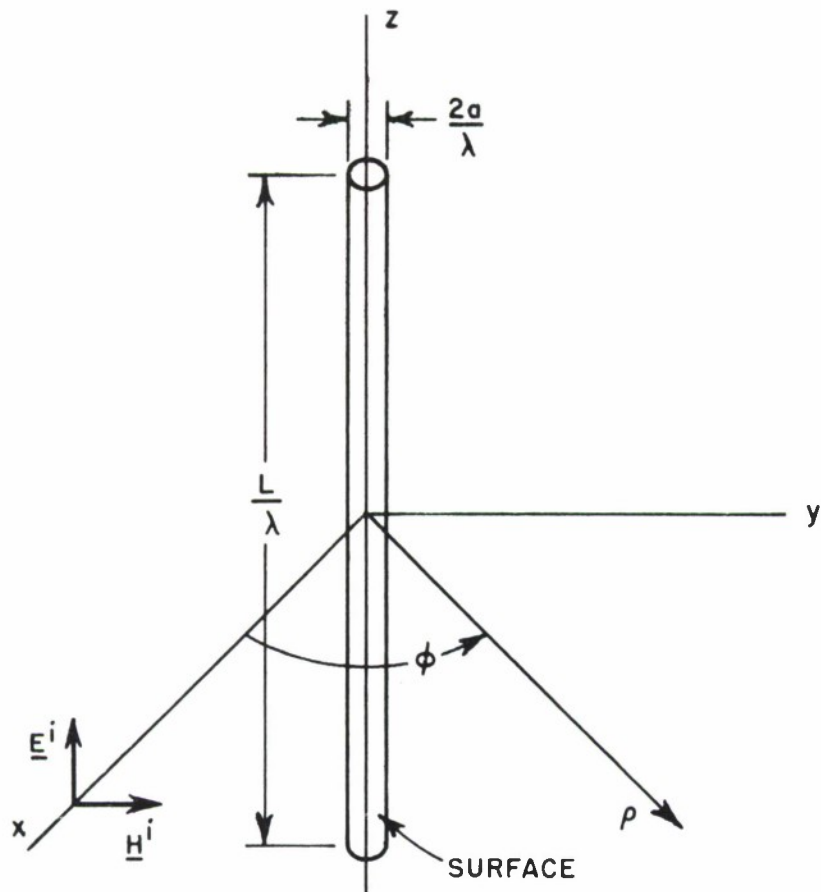


Fig. 1 Geometry and coordinate system associated with a thin linear dipole.

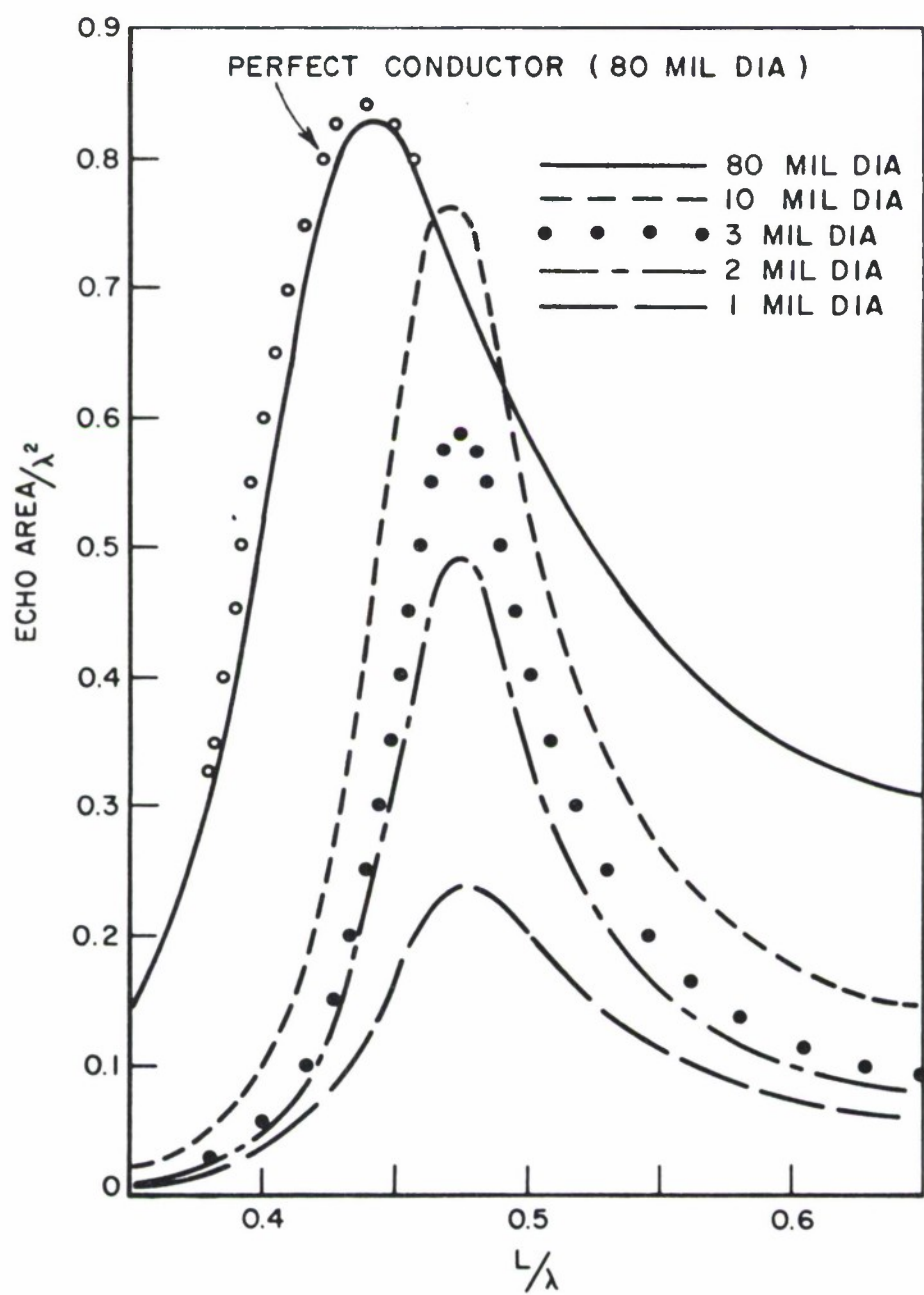


Fig. 2 Resonance curves for bismuth dipoles of various diameters at 3000 MHz (broadside aspect).



Table I  
Conductivities of Some Common Metals

Material	Conductivity	
Silver	60	MS/m
Copper	57	MS/m
Aluminum	35	MS/m
Tungsten	17.5	MS/m
Platinum	9.25	MS/m
Bismuth	0.815	MS/m

## II. THEORY

To obtain an expression for the total electric field in the dipole of Fig. 1, the scalar wave equation may be employed. That is,

$$(1) \quad (\nabla^2 + k^2)\psi = 0 \quad .$$

Using the method of separation of variables, we seek to find a solution having the form

$$(2) \quad \psi = R(\rho)\Phi(\phi)Z(z)$$

which implies that the radial variation of the tangential electric field, for instance, is independent of the z-variation. For thin dipoles, such as those we wish to consider here, this is a valid assumption. For purposes of definition, a thin dipole may be interpreted as being one whose radius is less than about  $0.01\lambda$ . Also, since the dipole is thin,  $\psi$  may be assumed to be independent of  $\phi$ . Thus,

$$(3) \quad \psi = R(\rho)Z(z) \quad .$$

Following through the solution of Eq. (1) by Eq. (3), one can obtain the following expression for z-component of the total electric field in the dipole,

$$(4) \quad E_z(\rho, z) = I_0(\gamma\rho) \sum_{n=1}^N E_n \cos \frac{(2n-1)\pi z}{L}$$

where

$$(5) \quad \gamma = \sqrt{j\omega\mu\sigma} \quad ,$$

and  $I_0(\gamma\rho)$  is the modified Bessel function of the first kind and zero order. The expression for  $\gamma$  is valid under the usual assumption that  $100 < \sigma/\omega\epsilon_0$ .

Evaluating Eq. (4) on the axis of the dipole (ie.,  $\rho = 0$ ) and at the surface (ie.,  $\rho = a$ ) and taking the ratio of the magnitudes, one obtains

$$(6) \quad \frac{|E_z(\rho=0)|}{|E_z(\rho=a)|} = \frac{1}{|I_0(\gamma a)|}$$

since the modified Bessel function of zero order having zero argument is unity. Thus, the penetration of the electric field to the dipole axis is solely dependent on the magnitude of  $I_0(\gamma a)$ . This implies that the maximum or peak echo area, which is sensitive to the electric field penetration, may also be related to this simple function. It is to this task that we now turn our attention.

### III. CALCULATED RESULTS

Shown in Figs. 3a and 4a are curves of the maximum echo area/ $\lambda^2$  versus the dipole diameter in mils for bismuth and platinum respectively. These curves show that the maximum echo area declines as the physical wire diameter decreases or as the conductivity is lowered. Since the resonant length of a dipole is dependent on the diameter of the wire used, the values obtained for the maximum echo area were calculated using the resonant length for each wire diameter. Consequently, it was necessary to compute the top portion of many resonance curves before plotting Figs. 3a and 4a. The computation of these curves, and those of Fig. 2, was accomplished using the point-matching technique [1-4].

Illustrated in Figs. 3b and 4b are curves illustrating the penetration of the electric field into the dipoles whose maximum echo areas are given in Figs. 3a and 4a. A comparison of Figs. 3a and 3b and also of 4a and 4b shows that, as the electric field penetrates to the dipole axis with significant strength, the echo area decreases rather markedly. This decrease in the echo area continues toward zero as the electric field on axis approaches a value similar to that on the surface at  $\rho = a$ . When this occurs, the imperfectly conducting dipole may be thought of as acting like a dielectric rod.

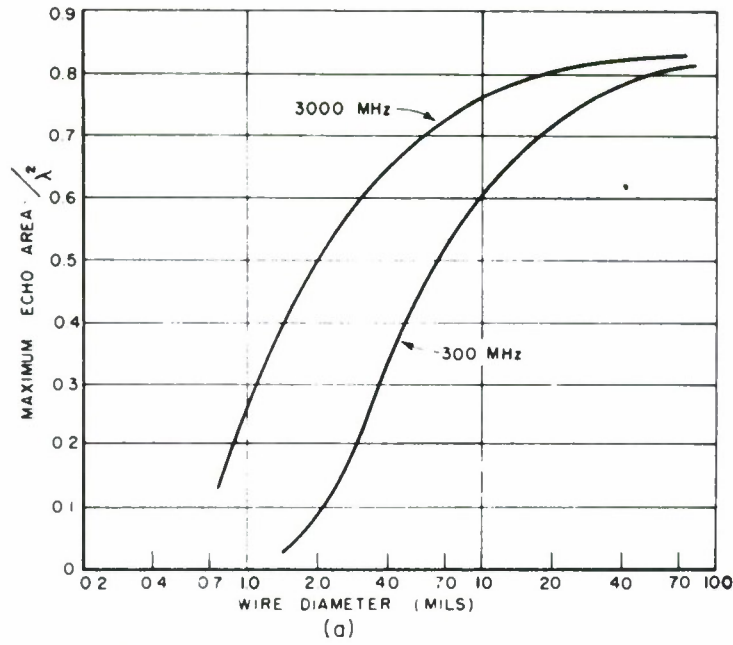


Fig. 3a Dependence of the maximum echo area of bismuth dipoles on the dipole diameter.

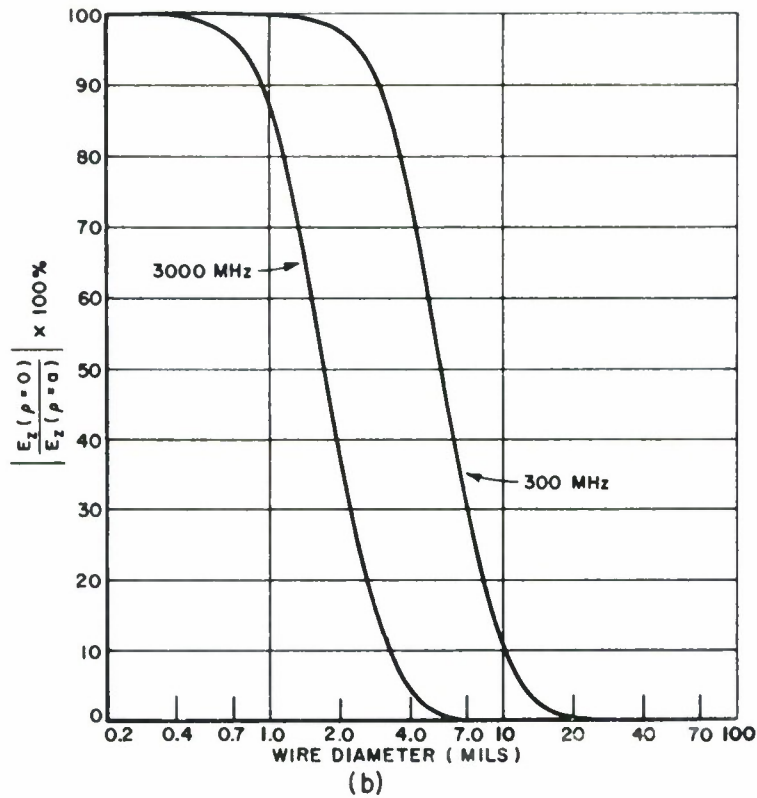


Fig. 3b Penetration of the electric field into bismuth dipoles of various diameters.

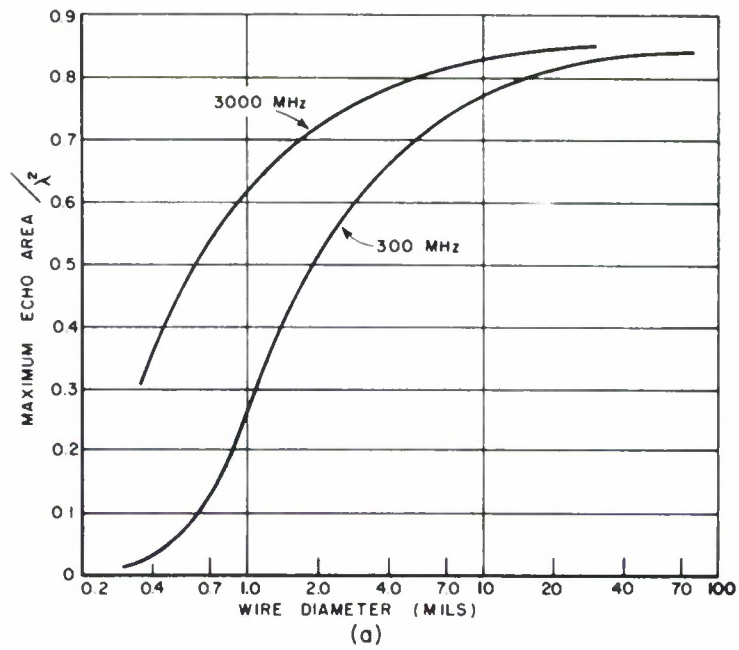


Fig. 4a Dependence of the maximum echo area of platinum dipoles on the dipole diameter.

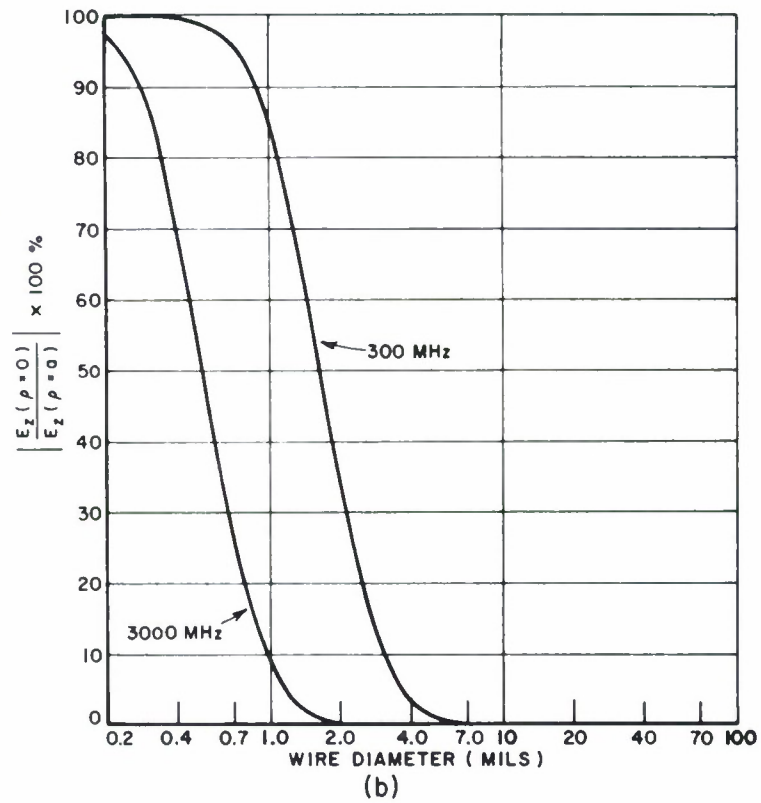


Fig. 4b Penetration of the electric field into platinum dipoles of various diameters.



From the four sets of curves plotted in Figs. 3 and 4 it was observed that, for a given maximum echo area, the ratio of  $|E_z(\rho = 0)|/|E_z(\rho = a)|$  was essentially the same. As a direct result of this observation, the curve in Fig. 5 was deduced which relates the maximum echo area to the penetration of the electric field. Thus, Fig. 5 may be used to determine the maximum echo area of a dipole resonant at frequency  $\omega$  and having conductivity  $\sigma$  and radius  $a$ , simply by computing  $|I_0(\gamma a)|^{-1}$  or just  $|\gamma a|$ .

As a result of the above, it is now possible to offer the following hypothesis.

#### HYPOTHESIS

A thin imperfectly conducting linear dipole of conductivity  $\sigma_1$ , permeability  $\mu_1$ , having radius  $a_1$  and first resonance at frequency  $\omega_1$  will have the same maximum or peak echo area as a second resonant dipole ( $\sigma_2, \mu_2, a_2, \omega_2$ ) if  $100 < \sigma/\omega\epsilon_0$ ,  $a < 0.01\lambda$  and the condition is satisfied that

$$(7) \quad \gamma_1 a_1 = \gamma_2 a_2$$

or

$$(8) \quad \sqrt{j\omega_1 \mu_1 \sigma_1} a_1 = \sqrt{j\omega_2 \mu_2 \sigma_2} a_2$$

and conversely.

As a random check on the validity of Eq. (8) and the curve in Fig. 5, the actual echo area and also the penetration of the electric field was calculated for copper dipoles at 100 MHz and 3000 MHz, for tungsten dipoles at 1000 MHz, and for bismuth dipoles at 10 GHz. In all cases, agreement with Fig. 5 was very good.

#### IV SUMMARY AND CONCLUSIONS

In summary, curves have been presented showing the dependence of the maximum or peak echo area upon conductivity, frequency and dipole diameter. Corresponding curves showing the penetration of the electric field into the dipole are also given. From these curves, a general curve is deduced which relates the maximum echo area to the penetration of the electric field. An hypothesis is then given that specifies the conditions under which two different imperfectly conducting dipoles will have the same maximum echo area.

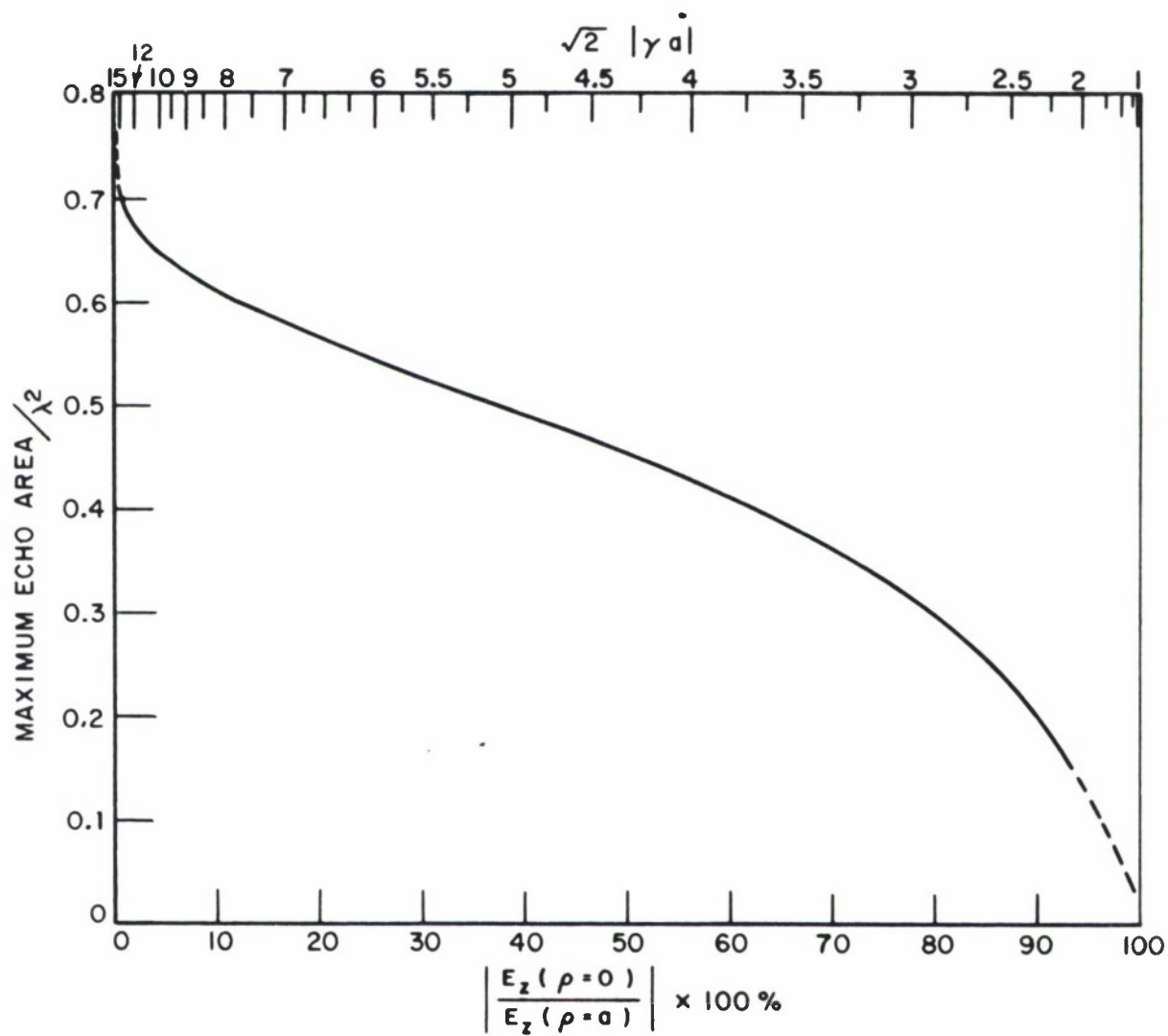


Fig. 5 Relationship of the maximum echo area to the penetration by the electric field.

Thus, we may conclude that it is now possible to determine the maximum echo area of imperfectly conducting dipoles without recourse to actual calculation of resonance curves. To do this, one uses the general curve mentioned above. This general curve should find use in situations where one is interested in studying the effect of various materials and wire diameters on the maximum echo area, the weight and the number of dipoles per unit volume as is often done in chaff studies. In addition, this curve along with the hypothesis serves to answer the question as to when the effect of imperfect conductivity on the echo area of a wire scatterer is significant and when it is not.

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